

Transverse spin and transverse momenta in hard scattering processes

P.J. Mulders

National Institute for Nuclear Physics and High-Energy Physics (NIKHEF)
P.O. Box 41882, NL-1009 DB Amsterdam, the Netherlands
and
Department of Physics and Astronomy, Free University
De Boelelaan 1081, NL-1081 HV Amsterdam, the Netherlands

October 1995
NIKHEF 95-057
hep-ph/9510317

Talk presented at the workshop on Prospects of Spin Physics at HERA, 28-31 August 1995,
DESY (Hamburg), Germany

Transverse spin and transverse momenta in hard scattering processes

P.J. Mulders

*National Institute for Nuclear Physics and High-Energy Physics
(NIKHEF), P.O. Box 41882, NL-1009 DB Amsterdam, the Netherlands*
and

*Department of Physics and Astronomy, Free University
De Boelelaan 1081, NL-1081 HV Amsterdam, the Netherlands*

Abstract

Inclusive and semi-inclusive deep inelastic leptonproduction offers possibilities to study details of the quark and gluon structure of the hadrons involved. In many of these experiments polarization is an essential ingredient. We also emphasize the dependence on transverse momenta of the quarks, which leads to azimuthal asymmetries in the produced hadrons.

1 Introduction

Hard processes using electroweak probes are very well suited to probe the quark and gluon structure of hadrons. The leptonic part is known, determining the kinematics of the electroweak probe. Examples of such processes are

- Lepton-hadron scattering (DIS)

$$\gamma^*(q) + H \longrightarrow h + X \quad (-q^2 \equiv Q^2 \geq 0)$$

- Drell-Yan scattering (DY)

$$H_A + H_B \longrightarrow \gamma^*(q) + X \quad (q^2 \equiv Q^2 \geq 0)$$

- Electron-positron annihilation

$$\gamma^*(q) \longrightarrow h_1 + h_2 + X \quad (q^2 \equiv Q^2 \geq 0)$$

The interaction of the electroweak probe with quarks is known.

We consider deep inelastic processes where Q is considerably larger (how much is mostly an empirical fact) than the typical hadronic scale Λ , which is of order 1 GeV. The large momentum Q makes it feasible to do the calculation within the framework of QCD. One writes down a diagrammatic expansion of the hard scattering amplitude (actually the squared amplitude), dividing it into hard and soft parts. The simplest (parton model diagram) for semi-inclusive ℓH scattering is shown in Fig. 1. The photon couples into the hard part, containing quark and

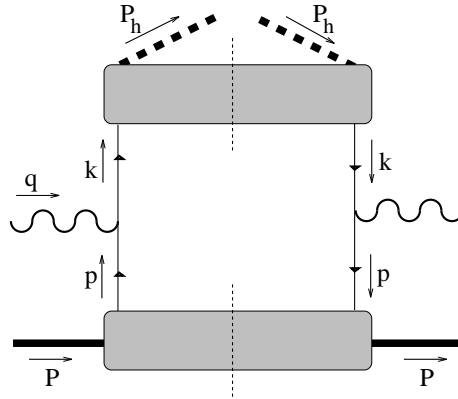


Figure 1: The parton level diagram for semi-inclusive deep inelastic scattering

gluon lines, while hadrons couple into soft parts, represented by a blob connecting hadron lines and quark and gluon lines for which the momenta satisfy $p_i \cdot p_j \sim \Lambda^2 \ll Q^2$. For the calculation of the hard part one can use the QCD Feynman rules, while for the soft parts simply the definition enters, being expectation values of quark and gluon fields in hadron states.

It turns out that at tree level the leading diagrams contain soft parts that are quark-quark correlation functions of the type shown in Fig. 2, given by [1, 2, 3]

$$\Phi_{ij}(p, P, S) = \frac{1}{(2\pi)^4} \int d^4x e^{ip \cdot x} \langle P, S | \bar{\psi}_j(0) \psi_i(x) | P, S \rangle, \quad (1)$$

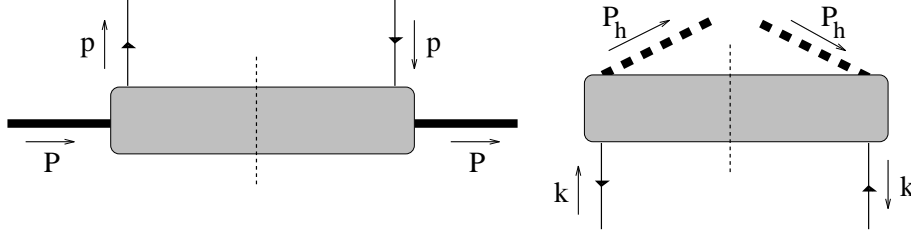


Figure 2: Quark-quark correlation function giving quark distributions (left) and fragmentation functions (right)

where a summation over color indices is implicit, and

$$\Delta_{ij}(k, P_h, S_h) = \sum_X \frac{1}{(2\pi)^4} \int d^4x e^{ik \cdot x} \langle 0 | \psi_i(x) | P_h, S_h; X \rangle \langle P_h, S_h; X | \bar{\psi}_j(0) | 0 \rangle \quad (2)$$

where an averaging over color indices is implicit. In both definitions flavor indices are suppressed and also the path ordered link operator needed to make the bilocal matrix element color gauge-invariant is omitted.

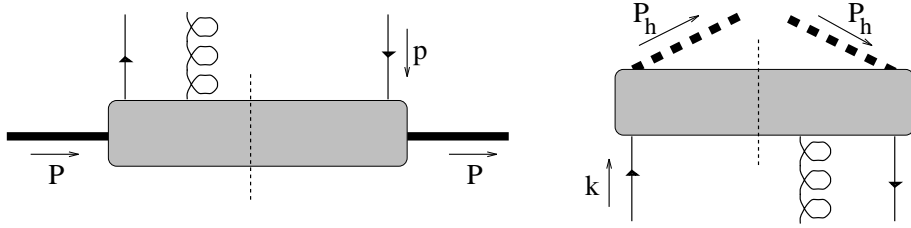


Figure 3: Quark-quark-gluon correlation functions contributing in hard scattering processes at subleading order.

The large scale Q leads to an ordering of the terms in the diagrammatic expansion [4] in powers of $1/Q$, α_s and $\alpha_s \ln Q^2$. Writing down the simplest diagram where a photon is absorbed on a quark one ends up with the combination of soft parts in Fig. 2. Gluonic corrections in the hard QCD part of the process can be absorbed in a scale dependence of the soft parts, at least at leading order (factorization). At order $1/Q$ also quark-quark-gluon correlation functions (shown in Fig. 3) appear. These can be rewritten in quark-quark correlation functions using the QCD equations of motion, provided that one does include the dependence on the transverse momenta of the quarks.

Next step is the analysis of the correlation functions including the transverse momentum dependence [5, 6]. It is convenient to parametrize the momenta in terms of lightcone coordinates, $p = [p^-, p^+, \mathbf{p}_T]$ with $p^\pm = (p^0 \pm p^3)/\sqrt{2}$. Choosing a frame in which the hadrons are collinear one writes for the hadrons and virtual photon in ℓH scattering,

$$P = \left[\frac{x_B M^2}{A\sqrt{2}}, \frac{A}{x_B\sqrt{2}}, \mathbf{0}_T \right] \equiv \frac{Q}{x_B\sqrt{2}} n_+ + \frac{x_B M^2}{Q\sqrt{2}} n_-, \quad (3)$$

$$P_h = \left[\frac{z_h Q^2}{A\sqrt{2}}, \frac{A M_h^2}{z_h Q^2 \sqrt{2}}, \mathbf{0}_T \right] \equiv \frac{z_h Q}{\sqrt{2}} n_- + \frac{M_h^2}{z_h Q\sqrt{2}} n_+, \quad (4)$$

$$q = \left[\frac{Q^2}{A\sqrt{2}}, -\frac{A}{\sqrt{2}}, \mathbf{q}_T \right] = \frac{Q}{\sqrt{2}} n_+ - \frac{Q}{\sqrt{2}} n_- + q_T. \quad (5)$$

Note that in a frame in which P and q have no transverse momenta, the outgoing hadron has a transverse momentum $\mathbf{P}_{h\perp} = -z\mathbf{q}_T$. The calculation of the diagrams involves an integral over soft parts,

$$\Phi^{[\Gamma]}(x, \mathbf{p}_T) = \frac{1}{2} \int dp^- \text{Tr}(\Phi \Gamma) \Big|_{p^+ = xP^+, \mathbf{p}_T}, \quad (6)$$

$$\Delta^{[\Gamma]}(z, \mathbf{k}_T) = \frac{1}{4z} \int dk^+ \text{Tr}(\Delta \Gamma) \Big|_{k^- = P_h^-/z, \mathbf{k}_T}. \quad (7)$$

Depending on the Dirac matrix Γ , these correlation functions are parametrized in terms of distribution and fragmentation functions, e.g. for a polarized spin 1/2 target with spin vector $S = [-\lambda M/2P^+, \lambda P^+/M, \mathbf{S}_T]$ with $\lambda^2 + \mathbf{S}_T^2 = 1$,

$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T) = f_1(x, \mathbf{p}_T), \quad (8)$$

$$\Phi^{[\gamma^+\gamma_5]}(x, \mathbf{p}_T) = \lambda g_{1L}(x, \mathbf{p}_T) + g_{1T}(x, \mathbf{p}_T) \frac{(\mathbf{p}_T \cdot \mathbf{S}_T)}{M} \equiv g_{1s}(x, \mathbf{p}_T), \quad (9)$$

$$\Phi^{[i\sigma^{+i}\gamma_5]}(x, \mathbf{p}_T) = S_T^i h_{1T}(x, \mathbf{p}_T) + \frac{p_T^i}{M} h_{1s}^\perp(x, \mathbf{p}_T), \quad (10)$$

$$\Phi^{[1]}(x, \mathbf{p}_T) = \frac{M}{P^+} e(x, \mathbf{p}_T) \quad (11)$$

$$\Phi^{[\gamma^i]}(x, \mathbf{p}_T) = \frac{p_T^i}{P^+} f^\perp(x, \mathbf{p}_T), \quad (12)$$

$$\Phi^{[\gamma^i\gamma_5]}(x, \mathbf{p}_T) = \frac{M S_T^i}{P^+} g'_T(x, \mathbf{p}_T) + \frac{p_T^i}{P^+} g_s^\perp(x, \mathbf{p}_T) \quad (13)$$

$$\Phi^{[i\sigma^{ij}\gamma_5]}(x, \mathbf{p}_T) = \frac{S_T^i p_T^j - p_T^i S_T^j}{P^+} h_T^\perp(x, \mathbf{p}_T) \quad (14)$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}(x, \mathbf{p}_T) = \frac{M}{P^+} h_s(x, \mathbf{p}_T). \quad (15)$$

In naming the functions we have extended the scheme proposed by Jaffe and Ji [7] for the \mathbf{k}_T -integrated functions. Depending on the Lorentz structure of the Dirac matrices Γ the parametrization involves powers $(1/P^+)^{t-2}$, where t is referred to as 'twist'. Integrated over k_T and taking moments in x it corresponds to the OPE 'twist' of the (in that case) local operators. When everything is done it will turn out that the factors $1/P^+$ give rise to factors $1/Q$ in the cross sections. The leading projections $\Phi^{[\gamma^+]}$, $\Phi^{[\gamma^+\gamma_5]}$ and $\Phi^{[i\sigma^{+i}\gamma_5]}$ can be interpreted as quark momentum densities, namely the unpolarized distribution, the chirality (for massless quarks helicity) distribution and the transverse spin distribution, respectively.

For the fragmentation functions one has an analogous analysis, which for unpolarized final state hadrons yields

$$\Delta^{[\gamma^-]}(z, \mathbf{k}_T) = D_1(z, -z\mathbf{k}_T), \quad (16)$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z, \mathbf{k}_T) = \frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp(z, -z\mathbf{k}_T), \quad (17)$$

$$\Delta^{[1]}(z, \mathbf{k}_T) = \frac{M_h}{P_h^-} E(z, -z\mathbf{k}_T), \quad (18)$$

$$\Delta^{[\gamma^i]}(z, \mathbf{k}_T) = \frac{k_T^i}{P_h^-} D^\perp(z, -z\mathbf{k}_T), \quad (19)$$

$$\Delta^{[i\sigma^{ij}\gamma_5]}(z, \mathbf{k}_T) = \frac{M_h \epsilon_T^{ij}}{P_h^-} H(z, -z\mathbf{k}_T). \quad (20)$$

As indicated before, the twist-3 function in $\Phi^{[\gamma^i \gamma_5]}$ surviving after k_T -integration, $g_T = g'_T + (\mathbf{k}_T^2/2M^2) g_T^\perp$, appears at subleading order. Reinstating the summation over quark flavors and identifying the result with the most general cross sections, expressed in terms of structure functions, one obtains

$$\frac{F_2(x_B, Q^2)}{x_B} = 2 F_1(x_B, Q^2) = \sum_a e_a^2 \left(f_1^{(a)}(x_B) + \bar{f}_1^{(a)}(x_B) \right), \quad (22)$$

$$2 g_1(x_B, Q^2) = \sum_a e_a^2 \left(g_1^{(a)}(x_B) + \bar{g}_1^{(a)}(x_B) \right), \quad (23)$$

$$g_T(x_B, Q^2) = g_1(x_B, Q^2) + g_2(x_B, Q^2) = \frac{1}{2} \sum_a e_a^2 \left(g_T^{(a)}(x_B) + \bar{g}_T^{(a)}(x_B) \right). \quad (24)$$

The second example is semi-inclusive scattering including the dependence on the transverse momentum $\mathbf{P}_{h\perp}$ of the detected hadron [9, 10]. For this we assume a gaussian transverse momentum dependence for the quark distribution and fragmentation functions,

$$f(x, \mathbf{p}_T^2) = f(x) \frac{R_H^2}{\pi} \exp(-R_H^2 \mathbf{p}_T^2) \equiv f(x) \mathcal{G}(|\mathbf{p}_T|; R_H), \quad (25)$$

$$D(z, z^2 \mathbf{k}_T^2) = D(z) \frac{R_h^2}{\pi z^2} \exp(-R_h^2 \mathbf{k}_T^2) = \frac{D(z)}{z^2} \mathcal{G}(|\mathbf{k}_T|; R_h) = D(z) \mathcal{G}\left(z|\mathbf{k}_T|; \frac{R_h}{z}\right). \quad (26)$$

This enables us to express the results in the \mathbf{p}_T -integrated distributions and a (normalized) gaussian distribution, while we can evaluate the complex-looking convolutions in transverse momenta that appear in the cross sections replacing them by a simple gaussian distribution in Q_T . The result for the cross section is

$$\begin{aligned} \frac{d\sigma}{dx_B dy dz_h d^2 \mathbf{P}_{h\perp}} &= \frac{4\pi\alpha^2 s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left[\frac{y^2}{2} + 1 - y \right] x_B f_1^a(x_B) D_1^a(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2} \\ &\quad - \frac{4\pi\alpha^2 s}{Q^4} \lambda \sum_{a,\bar{a}} e_a^2 (1 - y) \sin(2\phi_h) \frac{Q_T^2 R^4}{M M_h R_H^2 R_h^2} x_B h_{1L}^{\perp a}(x_B) H_1^{\perp a}(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2} \\ &\quad - \frac{4\pi\alpha^2 s}{Q^4} |\mathbf{S}_\perp| \sum_{a,\bar{a}} e_a^2 \left\{ (1 - y) \sin(\phi_h + \phi_s) \frac{Q_T R^2}{M_h R_h^2} x_B h_1^a(x_B) H_1^{\perp a}(z_h) \right. \\ &\quad \left. + (1 - y) \sin(3\phi_h - \phi_s) \frac{Q_T^3 R^6}{2M^2 M_h R_H^4 R_h^2} x_B h_{1T}^{\perp a}(x_B) H_1^{\perp a}(z_h) \right\} \frac{\mathcal{G}(Q_T; R)}{z_h^2} \\ &\quad + \frac{4\pi\alpha^2 s}{Q^4} \lambda_e \lambda \sum_{a,\bar{a}} e_a^2 y \left(1 - \frac{y}{2} \right) x_B g_{1L}^a(x_B) D_1^a(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2} \\ &\quad + \frac{4\pi\alpha^2 s}{Q^4} |\mathbf{S}_\perp| \sum_{a,\bar{a}} e_a^2 y \left(1 - \frac{y}{2} \right) \cos(\phi_h - \phi_s) \frac{Q_T R^2}{M R_H^2} g_{1T}^a(x_B) D_1^a(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2}. \end{aligned} \quad (27)$$

We see that all six twist-two x - and \mathbf{p}_T -dependent quark distribution functions for a spin 1/2 hadron can be accessed in leading order asymmetries if one considers lepton and hadron polarizations. One of the asymmetries involves the transverse spin distribution h_1^a [11]. On the production side, only two different fragmentation functions are involved, the familiar unpolarized fragmentation function D_1^a and the fragmentation function $H_1^{\perp a}$. The latter is one of the functions which depends on interactions and is allowed in the fragmentation process because one cannot use time-reversal invariance.

As our last example, we give the extension of the above result up to order $1/Q$ for an unpolarized nucleon target. One obtains

$$\frac{d\sigma}{dx_B dy dz_h d^2 \mathbf{P}_{h\perp}} = \frac{4\pi\alpha^2 s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left\{ \left[\frac{y^2}{2} + 1 - y \right] x_B f_1^a(x_B) D_1^a(z_h) \right.$$

$$\begin{aligned}
& - 2(2-y)\sqrt{1-y} \cos(\phi_h) \frac{Q_T}{Q} \left(\frac{R^2}{R_H^2} x_B^2 f^{\perp a}(x_B) D_1^a(z_h) - \frac{R^2}{R_h^2} x_B f_1^a(x_B) \frac{\tilde{D}^{\perp a}(z_h)}{z_h} \right) \\
& - \lambda_e 2y\sqrt{1-y} \sin \phi_h \frac{Q_T}{Q} \frac{M R^2}{M_h R_h^2} x_B^2 \tilde{e}^a(x_B) H_1^{\perp a}(z_h) \Big\} \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \tag{28}
\end{aligned}$$

The $\langle \cos(\phi_h) \rangle$ asymmetry in unpolarized leptonproduction, unfortunately is rather complicated, involving one twist-three distribution function ($f^{\perp a}$) and one twist-three fragmentation function ($D^{\perp a}$) [12]. It is important to point out, however, that the $\langle \cos(\phi_h) \rangle$ asymmetry is not only a kinematical effect [13]. It reduces to a kinematical factor only depending on y and Q^2 when the interaction-dependent pieces in the twist-three functions [8] are set to zero, $\tilde{f}^{\perp a} = f^{\perp a} - f_1^a/x_B = 0$ and $\tilde{D}^{\perp a} = D^{\perp a} - z_h D_1^{\perp a} = 0$. At order $1/Q$ there is no $\langle \cos(2\phi_h) \rangle$ asymmetry in the deep-inelastic leptonproduction cross section. For polarized leptons and unpolarized targets a $\langle \sin(\phi_h) \rangle$ asymmetry is found [14], involving the interaction dependent part of the distribution function e^a , $\tilde{e}^a = e^a - (m/M)f_1^a$, and the time-reversal odd fragmentation function $H_1^{\perp a}$. Noteworthy is that it is the same fragmentation function that appears in several of the leading azimuthal asymmetries for polarized targets.

This work is part of the research program of the foundation for Fundamental Research of Matter (FOM) and the National Organization for Scientific Research (NWO).

References

- [1] D.E. Soper, Phys. Rev. D **15** (1977) 1141; Phys. Rev. Lett. **43** (1979) 1847
- [2] J.C. Collins and D.E. Soper, Nucl. Phys. **B194** (1982) 445
- [3] R.L. Jaffe, Nucl. Phys. **B229** (1983) 205
- [4] R.K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. **B212** (1983) 29
- [5] J. P. Ralston and D. E. Soper, Nucl. Phys. B **152** (1979) 109
- [6] R. D. Tangerman and P. J. Mulders, Phys. Rev. D **51** (1995) 3357
- [7] R.L. Jaffe and X. Ji, Nucl. Phys. **B375** (1992) 527
- [8] P.J. Mulders and R.D. Tangerman, NIKHEF report 95-053, hep-ph/9510301
- [9] A. Kotzinian, Nucl. Phys. B **441** (1995) 234
- [10] R. D. Tangerman and P. J. Mulders, Phys. Lett. B **352** (1995) 129
- [11] J. Collins, Nucl. Phys. **B396** (1993) 161
- [12] J. Levelt and P.J. Mulders, Phys. Rev. **D49** (1994) 96
- [13] R. Cahn, Phys. Lett. **78** (1978) 269
- [14] J. Levelt and P.J. Mulders, Phys. Lett. B **338** (1994) 357